Mech 510

Computational Methods in Transport Phenomena I

Mech 510 is a first course in CFD from the code development point of view. The goal of the course is to introduce you to techniques for analyzing the accuracy and stability properties of finite volume schemes for solving partial differential equations and to writing and debugging programs that implement some of these schemes. For many of you, this will be a prelude to more complex CFD development tasks later in your graduate studies; for the rest, understanding the details behind how CFD schemes work will make you more knowledgable and proficient users of existing CFD codes.

Instructor: Carl Ollivier-Gooch

My research interests are in algorithms for flow solution on unstructured meshes, algorithms for unstructured mesh generation, and aerodynamic shape optimization. I'm happy to discuss any of these topics (or just about anything else related to CFD or fluids in general) at length.

Marking:

25%	Three homework assignments.
35%	Three programming assignments.
40%	Final project (due during the final exam period).

Recommended Reading: None of these texts are required, but they are all good books on CFD. There are others as well, of course, but the first two I can personally testify to the quality of, and the third is widely endorsed by others.

Lomax, Pulliam and Zingg, Fundamentals of Computational Fluid Dynamics. Second Edition, 2003.

Hirsch, Numerical Computation of Internal and External Flows, Vols. 1 and 2, 1990.

Ferziger and Peric, Computational Methods for Fluid Dynamics, 1997.

Course Outline

Introduction. Comparison of strengths of analytical, computational, and experimental methods in fluid mechanics. Example of how one works through from a physical problem to a working, trustworthy program to solve that problem. Derivation of simple scalar model equations for the Navier-Stokes equations.

Space discretization techniques. Comparison of finite difference, finite element, and finite volume approaches to discretization of a model problem. Detailed basis of the finite volume method, including construction of the integral form, flux identification, and flux computation. Accuracy analysis of finite volume fluxes and flux integrals. Accuracy assessment based on numerical results.

Time discretization and stability. Time accuracy analysis for ODE's and systems of ODE's. Stability analysis for time advance and for PDE's. Explicit and implicit time advance methods.

Finite-volume discretization of Poisson's equation. Flux evaluation for the Laplacian operator. Boundary conditions. Direct and iterative schemes for Laplace's equation in two dimensions.

Finite-volume discretization of the wave equation. Evaluation of non-gradient fluxes. Upwinding. Boundary conditions, including numerical boundary conditions. Explicit time advance schemes applied to the wave equation.

Application problem: Incompressible energy equation. Discretization in space and time. Time advance methods. Boundary conditions. Validation.

Application problem: Laminar Navier-Stokes equations. Overview of discretization methods for incompressible Navier-Stokes. Computational formulation of the equations. Discretization in time and space. Boundary conditions. Implicit time advance schemes for systems of equations. Validation techniques for CFD codes.

Programming

This is a ***computational*** fluid dynamics course, and with many parts of CFD, you have to program it to really understand it, so there will be a fairly large amount of programming required. If you aren't comfortable in some compiled language (C, Fortran, Pascal, or something similar), you should see me. Note that things like Matlab aren't acceptable as the finished product. While Matlab is an excellent tool for proof-of-concept stuff, it has limitations that make it impractical for large-scale CFD problems, so you need to learn how to code these things in a more efficient language.

Learning Objectives

This list of learning objectives describes the most important things that I expect students to be able to do by the end of the term. Accordingly, I encourage you to consult the list regularly to see where you stand and where you need to go.

By the end of the term, students should be able to:

- Convert a partial differential equation into control volume form by using Gauss' Theorem.
- Derive correct expressions for the flux through each face on a computational control volume for a governing equation in integral form.
- Analyze the spatial accuracy of a flux approximation and the time accuracy of a time advance scheme by using Taylor series analysis.
- Analyze the stability characteristics of a time advance scheme by using a model ordinary differential equation.
- Determine the eigenvalues of a spatial discretization scheme in one dimension with periodic boundary conditions.
- Combine eigenvalue analysis of a spatial discretization scheme with model equation analysis of a time advance scheme to determine the stability limitations of the combined space-time scheme.
- Discuss the advantages and disadvantages of explicit and implicit time advance schemes for the wave equation and heat equation.
- Compare the efficiency of two relaxation techniques for the Laplace equation.
- Given solutions to a computational problem obtained on a series of successively finer meshes, determine how well the solution on the finest mesh represents the exact solution to the mathematical model of the physical problem.
- Given the dependent variables and flux vector for a system of PDE's, compute the flux Jacobian. Likewise, compute the Jacobians for transforming from one set of variables to another.
- Describe an algorithm for solving the laminar incompressible Navier-Stokes equations.
- Write a computer program in a compiled language that will solve the laminar Navier-Stokes equations for a geometrically simple domain and describe the validation procedure used to determine that the program gives physically correct answers.